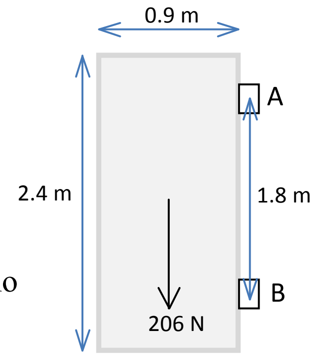
22. The bottle balances because all the clockwise torques are equal to the anticlockwise torques. Because the centre of mass of the bottle lies directly over the pivot it can exert no torque about the base (distance to the pivot is zero) and the system is therefore stable. The centre of mass of the bottle lying directly over the base is the critical design feature. Different sized bottles might not have their centre of mass lying outside the base and hence may topple because a net torque is produced.



23. a) Taking torques about upper hinge A:

$\Sigma ACT = \Sigma CT$

$206 \times 0.45 = 1.8\ F_B$

(for F_B to exert a clockwise torque it must act to the **left**)

$F_B = 51.5$ N horizontally to the left.

As all the weight is supported by hinge A, there will be no vertical force at B, so the net force at B is 51.5 N left.

b) To find F_A: Taking torques about lower hinge B:

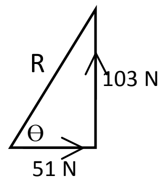
$\Sigma ACT = \Sigma CT$

$206 \times 0.45 = 1.8\ F_A$

(for F_A to exert a clockwise torque it must act to the **right**)

$F_A = 51.5$ N horizontally to the Right

Vertical force at A = ½ x 206 = 103 N up

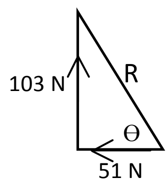


Total force = sum of vertical and horizontal forces (vector triangle)

$R^2 = 103^2 + 51^2$ so R = 115 N at $\tan^{-1} \Theta = 63.4^o$ right of horizontal.

To find F_B: The calculation is exactly the same as for F_A but the horizontal force will be towards the right.

Vector triangle answer is $F_B = 115$ N at $\tan^{-1} \Theta = 63.4^o$ left of horizontal.



24. a) Taking torques about the hinge:

$\Sigma CT = \Sigma ACT$

$(117.6 \times 0.8) + (441 \times 1.3) = T\sin20 \times 1.6$

$T = 1.22 \times 10^3$ N

b) Equating all the vertical forces on the beam:

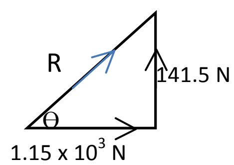
$117.6 + 441$ (down) $= 1.22 \times 10^3\sin20 + R_v$ (up)

$R_v = 141.5$ N up

Equating horizontal forces on the beam:

$R_H = 1.22 \times 10^3\cos20$

$R_H = 1.15 \times 10^3$ N to the right



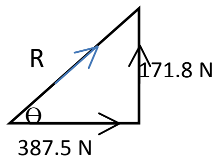
c) Total reaction force R is found from the vector triangle.

$R^2 = (1.15 \times 10^3)^2 + 141.5^2$ $R = 1.16 \times 10^3$ N at $\Theta = 7.01^o$ to the horizontal.

Motion and Forces in a Gravitational Field: Set 5

25. a) Taking torques about the hinge:
$\Sigma CT = \Sigma ACT$
$(343 \times 0.7) + (500 \times 1.4) = 1.4 \times T\sin60$
$T = 775$ N

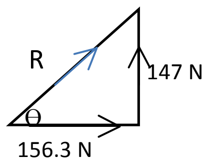
b) Equating all the vertical forces on the beam:
$343 + 500$ (down) $= 775\sin60 + R_v$ (up)
$R_v = 171.5$ N up
Equating all the horizontal forces on the beam:
$R_H = 775.4\cos60 = 387.7$ N (right).
Total R (vector diagram): $R^2 = 387.7^2 + 171.5^2$
$R = 424$ N at an angle of $\tan^{-1}$ so $\Theta = 23.9^o$



26. Let the maximum distance of the man from the pivot be d
Taking torques about the hinge: $\Sigma CT = \Sigma ACT$
$(490 \times 2) + (735 \times d) = 2.4 \times (1.36 \times 10^3\sin50)$
$735d = 2500 - 980$
$d = 2.07$ m.

27. a) Taking torques about the hinge: $\Sigma CT = \Sigma ACT$
$(49 \times 0.85\cos53.1) + (98 \times 1.7\cos53.1) = 0.8T$
$T = 156$ N

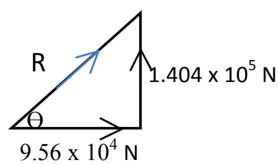
b) Equating all the vertical forces on the beam:
$R_v = 49 + 98 = 147$ N (up)
Equating all the horizontal forces on the beam:
$R_H = 156.3$ N (right)

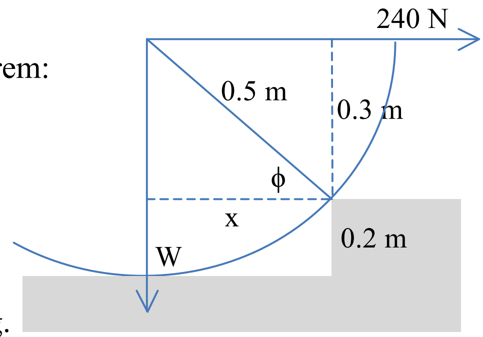


c) $R^2 = 156.3^2 + 147^2$
$R = 215$ N at an angle of $\tan^{-1}156.3$ so $\Theta = 43.2^o$

28. a) Let the length of the beam be L metres.
Taking torques about the hinge: $\Sigma CT = \Sigma ACT$
By geometry, top angle in the left triangle = 16.2^o
$[1.96 \times 10^4 \times 3] + [4.9 \times 10^4 \times L\cos53.1] = T \times L\sin16.2$
(L will cancel throughout) so $T = 1.20 \times 10^5$ N

b) Equating all the vertical forces on the beam:
$R_v = 1.96 \times 10^4 + 4.9 \times 10^4 + 1.195 \times 10^5\sin36.9$
$R_v = 1.40 \times 10^5$ N (up)
Equating all the horizontal forces on the beam:
$R_H = 1.20 \times 10^5\cos36.9 = 9.56 \times 10^4$ N (right)
$R^2 = (1.404 \times 10^5)^2 + (9.56 \times 10^4)^2$
$R = 1.70 \times 10^5$ N at an angle of $\tan^{-1}0.956$ so $\Theta = 55.7^o$





Q29. a) To find distance x we use Pythagoras' Theorem:

$x^2 = 0.50^2 - 0.30^2$ so $x = 0.40$ m

Taking torques about the step:

$\Sigma CT = \Sigma ACT$

$240 \times 0.3 = W \times 0.4$

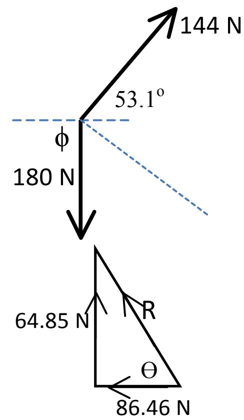
Hence W = 180 N mass = 180/9.8 = 18.4 kg.

b) Minimum force will occur when the distance to the pivot is a maximum, i.e. 0.5 m

Torques: $180 \times 0.4 = F \times 0.5$

$F_{min} = 144$ N

$\phi = \tan^{-1} 0.4 = 36.9^o$ Forces diagram →



Pulling angle to the horizontal = 90 – 36.87 = 53.1^0

Total vertical force = 180 - 144 sin53.1 = 64.85 down

Total horizontal force = 144cos53.1 = 86.46 right

Reaction force must oppose these two forces:

$R^2 = 64.85^2 + 86.46^2$

R = 108 N at $\tan^{-1}$86.46 so $\Theta = 36.9^o$ above the horizontal, left.